HEAT TRANSFER

There are three modes of heat transfer: conduction, convection, and radiation.

BASIC HEAT TRANSFER RATE EQUATIONS

Conduction

Fourier's Law of Conduction

$$\dot{Q} = -kA\frac{dT}{dx}\text{, where}$$

$\dot{Q}$ = rate of heat transfer (W)
k = the thermal conductivity [W/(m•K)]
A = the surface area perpendicular to direction of heat transfer (m^2)

Convection

Newton's Law of Cooling

$$\dot{Q} = hA\left(T_w - T_\infty\right)\text{, where}$$

h = the convection heat transfer coefficient of the fluid [W/(m^2•K)]
A = the convection surface area (m^2)
T_w = the wall surface temperature (K)
T_∞ = the bulk fluid temperature (K)

Radiation

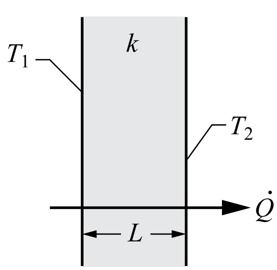
The radiation emitted by a body is given by

$$\dot{Q} = \varepsilon\sigma A T^4\text{, where}$$

ε = the emissivity of the body
σ = the Stefan-Boltzmann constant
= 5.67×10^{-8} W/(m^2•K^4)
A = the body surface area (m^2)
T = the absolute temperature (K)

CONDUCTION

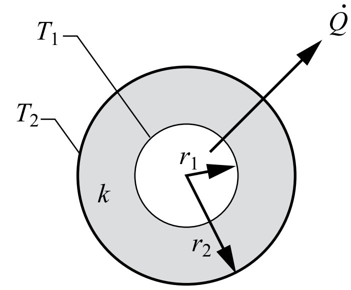
Conduction Through a Plane Wall



$$\dot{Q} = \frac{-kA\left(T_2 - T_1\right)}{L}\text{, where}$$

A = wall surface area normal to heat flow (m^2)
L = wall thickness (m)
T_1 = temperature of one surface of the wall (K)
T_2 = temperature of the other surface of the wall (K)

Conduction Through a Cylindrical Wall

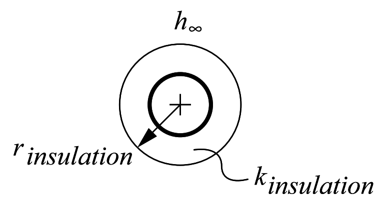


Cylinder (Length = L)

$$\dot{Q} = \frac{2\pi kL\left(T_1 - T_2\right)}{\ln\left(\frac{r_2}{r_1}\right)}$$

Critical Insulation Radius

$$r_{cr} = \frac{k_{insulation}}{h_\infty}$$



Thermal Resistance (R)

$$\dot{Q} = \frac{\Delta T}{R_{total}}$$

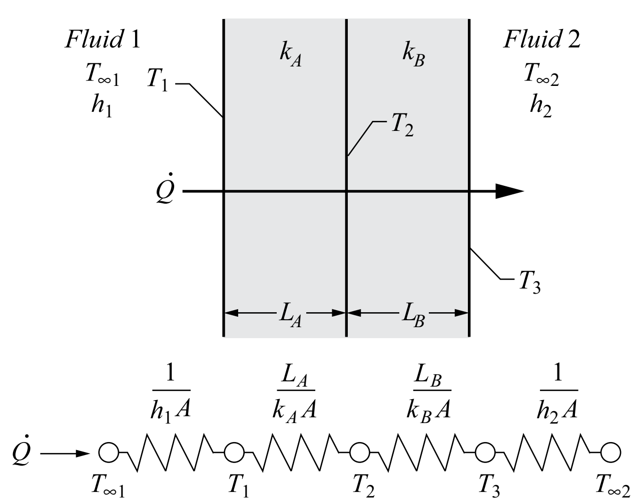
Resistances in series are added: $R_{total} = \Sigma R$, where

Plane Wall Conduction Resistance (K/W): $R = \frac{L}{kA}$, where
L = wall thickness

Cylindrical Wall Conduction Resistance (K/W): $R = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi kL}$, where
L = cylinder length

Convection Resistance (K/W) : $R = \frac{1}{hA}$

Composite Plane Wall



To evaluate Surface or Intermediate Temperatures:

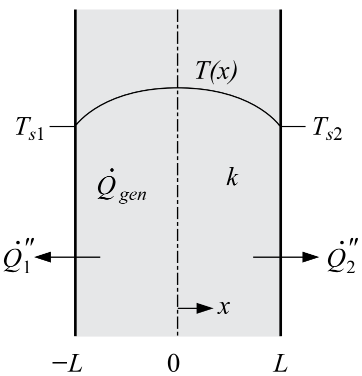
$$\dot{Q} = \frac{T_1 - T_2}{R_A} = \frac{T_2 - T_3}{R_B}$$

Steady Conduction with Internal Energy Generation
The equation for one-dimensional steady conduction is

$$\frac{d^2T}{dx^2} + \frac{\dot{Q}_{gen}}{k} = 0, \text{ where}$$

$\dot{Q}_{gen}$ = the heat generation rate per unit volume (W/m^3)

For a Plane Wall



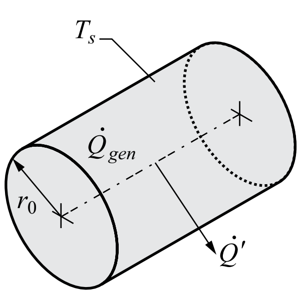
$$T(x) = \frac{\dot{Q}_{gen}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + \left(\frac{T_{s2} - T_{s1}}{2}\right)\left(\frac{x}{L}\right) + \left(\frac{T_{s1} - T_{s2}}{2}\right)$$

$$\dot{Q}_1'' + \dot{Q}_2'' = 2\dot{Q}_{gen}L, \text{ where}$$

$\dot{Q}''$ = the rate of heat transfer per area (heat flux) (W/m^2)

$$\dot{Q}_1'' = k\left(\frac{dT}{dx}\right)_{-L} \text{ and } \dot{Q}_2'' = k\left(\frac{dT}{dx}\right)_{L}$$

For a Long Circular Cylinder



$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{Q}_{gen}}{k} = 0$$

$$T(r) = \frac{\dot{Q}_{gen}r_0^2}{4k}\left(1 - \frac{r^2}{r_0^2}\right) + T_s$$

$$\dot{Q}' = \pi r_0^2 \dot{Q}_{gen}, \text{ where}$$

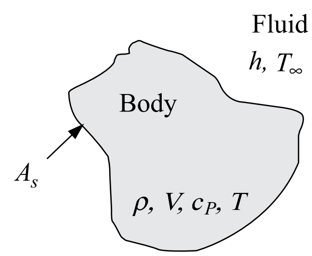
$\dot{Q}'$ = the heat transfer rate from the cylinder per unit length of the cylinder (W/m)

Transient Conduction Using the Lumped Capacitance Method

The lumped capacitance method is valid if

$$\text{Biot number, Bi} = \frac{hV}{kA_s} \ll 1, \text{ where}$$

h = the convection heat transfer coefficient of the fluid [W/(m^2•K)]
V = the volume of the body (m^3)
k = thermal conductivity of the body [W/(m•K)]
A_s = the surface area of the body (m^2)



Constant Fluid Temperature
If the temperature may be considered uniform within the body at any time, the heat transfer rate at the body surface is given by

$$\dot{Q} = hA_s(T - T_\infty) = -\rho V(c_P)\left(\frac{dT}{dt}\right), \text{ where}$$

T = the body temperature (K)
T_∞ = the fluid temperature (K)
ρ = the density of the body (kg/m^3)
c_P = the heat capacity of the body [J/(kg•K)]
t = time (s)

The temperature variation of the body with time is

$$T - T_\infty = (T_i - T_\infty)e^{-\beta t}, \text{ where}$$

$$\beta = \frac{hA_s}{\rho V c_P} \qquad \text{where } \beta = \frac{1}{\tau} \text{ and}$$

$$\tau = \text{time constant } (s)$$

The total heat transferred (Q_{total}) up to time t is

$$Q_{total} = \rho V c_P (T_i - T), \text{ where}$$

T_i = initial body temperature (K)

Variable Fluid Temperature

If the ambient fluid temperature varies periodically according to the equation

$$T_\infty = T_{\infty,\,mean} + \frac{1}{2}\left(T_{\infty,\,\max} - T_{\infty,\,\min}\right)\cos(\omega t)$$

The temperature of the body, after initial transients have died away, is

$$T = \frac{\beta\left[\frac{1}{2}\left(T_{\infty,\,\max} - T_{\infty,\,\min}\right)\right]}{\sqrt{\omega^2 + \beta^2}}\cos\left[\omega t - \tan^{-1}\left(\frac{\omega}{\beta}\right)\right] + T_{\infty,\,mean}$$

Fins

For a straight fin with uniform cross section (assuming negligible heat transfer from tip),

$$\dot{Q} = \sqrt{hPkA_c}\left(T_b - T_\infty\right)\tanh\left(mL_c\right), \text{ where}$$

h = the convection heat transfer coefficient of the fluid [W/(m^2•K)]

P = perimeter of exposed fin cross section (m)

k = fin thermal conductivity [W/(m•K)]

A_c = fin cross-sectional area (m^2)

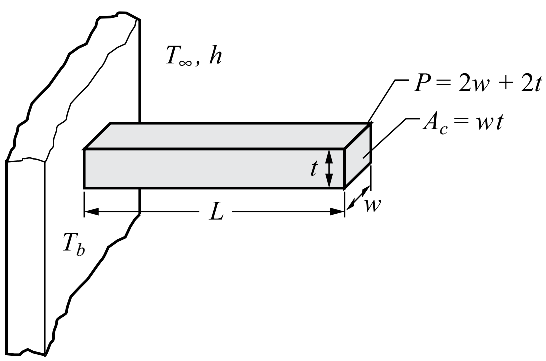
T_b = temperature at base of fin (K)

T_∞ = fluid temperature (K)

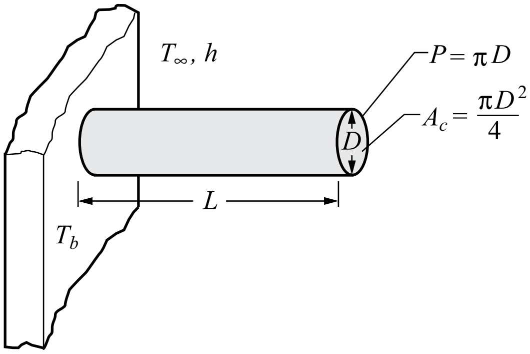
$m = \sqrt{\frac{hP}{kA_c}}$

$L_c = L + \frac{A_c}{P}$, corrected length of fin (m)

Rectangular Fin



Pin Fin



CONVECTION

Terms

D = diameter (m)

$\bar{h}$ = average convection heat transfer coefficient of the fluid [W/(m^2•K)]

L = length (m)

$\overline{Nu}$ = average Nusselt number

Pr = Prandtl number = $\frac{c_P\mu}{k}$

u_m = mean velocity of fluid (m/s)

u_∞ = free stream velocity of fluid (m/s)

μ = dynamic viscosity of fluid [kg/(s•m)]

ρ = density of fluid (kg/m^3)

External Flow

In all cases, evaluate fluid properties at average temperature between that of the body and that of the flowing fluid.

Flat Plate of Length L in Parallel Flow

$$\text{Re}_L = \frac{\rho u_\infty L}{\mu}$$

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.6640\,\text{Re}_L^{1/2}\,\text{Pr}^{1/3} \qquad \left(\text{Re}_L < 10^5\right)$$

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.0366\,\text{Re}_L^{0.8}\,\text{Pr}^{1/3} \qquad \left(\text{Re}_L > 10^5\right)$$

Cylinder of Diameter D in Cross Flow

$$\text{Re}_D = \frac{\rho u_\infty D}{\mu}$$

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = C\,\text{Re}_D^n\,\text{Pr}^{1/3}, \text{ where}$$

Re_D	C	n
1 – 4	0.989	0.330
4 – 40	0.911	0.385
40 – 4,000	0.683	0.466
4,000 – 40,000	0.193	0.618
40,000 – 250,000	0.0266	0.805

Flow Over a Sphere of Diameter, D

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 2.0 + 0.60\,\text{Re}_D^{1/2}\,\text{Pr}^{1/3},$$

$$\left(1 < \text{Re}_D < 70,000;\ 0.6 < \text{Pr} < 400\right)$$

Internal Flow

$$\text{Re}_D = \frac{\rho u_m D}{\mu}$$

Laminar Flow in Circular Tubes

For laminar flow (Re_D < 2300), fully developed conditions

Nu_D = 4.36 (uniform heat flux)

Nu_D = 3.66 (constant surface temperature)

For laminar flow ($\mathrm{Re}_D < 2300$), combined entry length with constant surface temperature

$$Nu_D = 1.86\left(\frac{\mathrm{Re}_D\mathrm{Pr}}{\frac{L}{D}}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}, \text{ where}$$

L = length of tube (m)

D = tube diameter (m)

μ_b = dynamic viscosity of fluid [kg/(s•m)] at bulk temperature of fluid, T_b

μ_s = dynamic viscosity of fluid [kg/(s•m)] at inside surface temperature of the tube, T_s

Turbulent Flow in Circular Tubes

For turbulent flow ($\mathrm{Re}_D > 10^4$, $\mathrm{Pr} > 0.7$) for either uniform surface temperature or uniform heat flux condition, Sieder-Tate equation offers good approximation:

$$Nu_D = 0.027\,\mathrm{Re}_D^{0.8}\mathrm{Pr}^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

Non-Circular Ducts

In place of the diameter, D, use the equivalent (hydraulic) diameter (D_H) defined as

$$D_H = \frac{4 \times \text{cross}-\text{sectional area}}{\text{wetted perimeter}}$$

Circular Annulus ($D_o > D_i$)

In place of the diameter, D, use the equivalent (hydraulic) diameter (D_H) defined as

$$D_H = D_o - D_i$$

Liquid Metals ($0.003 < \mathrm{Pr} < 0.05$)

$$Nu_D = 6.3 + 0.0167\,\mathrm{Re}_D^{0.85}\mathrm{Pr}^{0.93} \text{ (uniform heat flux)}$$

$$Nu_D = 7.0 + 0.025\,\mathrm{Re}_D^{0.8}\mathrm{Pr}^{0.8} \text{ (constant wall temperature)}$$

Condensation of a Pure Vapor

On a Vertical Surface

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.943\left[\frac{\rho_l^2 g h_{fg} L^3}{\mu_l k_l (T_{sat} - T_s)}\right]^{0.25}, \text{ where}$$

ρ_l = density of liquid phase of fluid (kg/m^3)

g = gravitational acceleration (9.81 m/s^2)

h_{fg} = latent heat of vaporization [J/kg]

L = length of surface [m]

μ_l = dynamic viscosity of liquid phase of fluid [kg/(s•m)]

k_l = thermal conductivity of liquid phase of fluid [W/(m•K)]

T_{sat} = saturation temperature of fluid [K]

T_s = temperature of vertical surface [K]

Note: Evaluate all liquid properties at the average temperature between the saturated temperature, T_{sat}, and the surface temperature, T_s.

Outside Horizontal Tubes

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 0.729\left[\frac{\rho_l^2 g h_{fg} D^3}{\mu_l k_l (T_{sat} - T_s)}\right]^{0.25}, \text{ where}$$

D = tube outside diameter (m)

Note: Evaluate all liquid properties at the average temperature between the saturated temperature, T_{sat}, and the surface temperature, T_s.

Natural (Free) Convection

Vertical Flat Plate in Large Body of Stationary Fluid

Equation also can apply to vertical cylinder of sufficiently large diameter in large body of stationary fluid.

$$\bar{h} = C\left(\frac{k}{L}\right)Ra_L^n, \text{ where}$$

L = the length of the plate (cylinder) in the vertical direction

Ra_L = Rayleigh Number = $\frac{g\beta(T_s - T_\infty)L^3}{v^2}\mathrm{Pr}$

T_s = surface temperature (K)

T_∞ = fluid temperature (K)

β = coefficient of thermal expansion (1/K)

(For an ideal gas: $\beta = \frac{2}{T_s + T_\infty}$ with T in absolute temperature)

υ = kinematic viscosity (m^2/s)

Range of Ra_L	C	n
$10^4 - 10^9$	0.59	1/4
$10^9 - 10^{13}$	0.10	1/3

Long Horizontal Cylinder in Large Body of Stationary Fluid

$$\bar{h} = C\left(\frac{k}{D}\right)\mathrm{Ra}_D^n, \text{ where}$$

$$\mathrm{Ra}_D = \frac{g\beta(T_s - T_\infty)D^3}{v^2}\mathrm{Pr}$$

Ra_D	C	n
$10^{-3} - 10^2$	1.02	0.148
$10^2 - 10^4$	0.850	0.188
$10^4 - 10^7$	0.480	0.250
$10^7 - 10^{12}$	0.125	0.333

Heat Exchangers

The rate of heat transfer in a heat exchanger is

$$\dot{Q} = UAF\Delta T_{lm}, \text{ where}$$

A = any convenient reference area (m^2)

F = heat exchanger configuration correction factor (F = 1 if temperature change of one fluid is negligible)

U = overall heat transfer coefficient based on area A and the log mean temperature difference [W/(m^2•K)]

ΔT_{lm} = log mean temperature difference (K)

Heat Exchangers (cont.)

Overall Heat Transfer Coefficient for Concentric Tube and Shell-and-Tube Heat Exchangers

$$\frac{1}{UA} = \frac{1}{h_iA_i} + \frac{R_{fi}}{A_i} + \frac{\ln\left(\frac{D_o}{D_i}\right)}{2\pi kL} + \frac{R_{fo}}{A_o} + \frac{1}{h_oA_o}, \text{ where}$$

A_i = inside area of tubes (m^2)
A_o = outside area of tubes (m^2)
D_i = inside diameter of tubes (m)
D_o = outside diameter of tubes (m)
h_i = convection heat transfer coefficient for inside of tubes [W/(m^2•K)]
h_o = convection heat transfer coefficient for outside of tubes [W/(m^2•K)]
k = thermal conductivity of tube material [W/(m•K)]
R_{fi} = fouling factor for inside of tube [(m^2•K)/W]
R_{fo} = fouling factor for outside of tube [(m^2•K)/W]

Log Mean Temperature Difference (LMTD)

For *counterflow* in tubular heat exchangers

$$\Delta T_{lm} = \frac{(T_{Ho} - T_{Ci}) - (T_{Hi} - T_{Co})}{\ln\left(\frac{T_{Ho} - T_{Ci}}{T_{Hi} - T_{Co}}\right)}$$

For *parallel flow* in tubular heat exchangers

$$\Delta T_{lm} = \frac{(T_{Ho} - T_{Co}) - (T_{Hi} - T_{Ci})}{\ln\left(\frac{T_{Ho} - T_{Co}}{T_{Hi} - T_{Ci}}\right)}, \text{ where}$$

ΔT_{lm} = log mean temperature difference (K)
T_{Hi} = inlet temperature of the hot fluid (K)
T_{Ho} = outlet temperature of the hot fluid (K)
T_{Ci} = inlet temperature of the cold fluid (K)
T_{Co} = outlet temperature of the cold fluid (K)

Heat Exchanger Effectiveness, ε

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{\text{actual heat transfer rate}}{\text{maximum possible heat transfer rate}}$$

$$\varepsilon = \frac{C_H(T_{Hi} - T_{Ho})}{C_{min}(T_{Hi} - T_{Ci})} \quad \text{or} \quad \varepsilon = \frac{C_C(T_{Co} - T_{Ci})}{C_{min}(T_{Hi} - T_{Ci})}$$

where
$C = \dot{m}c_P =$ heat capacity rate (W/K)
C_{min} = smaller of C_C or C_H

Number of Transfer Units (*NTU*)

$$NTU = \frac{UA}{C_{min}}$$

Effectiveness-NTU Relations

$$C_r = \frac{C_{min}}{C_{max}} = \text{heat capacity ratio}$$

For *parallel flow concentric tube* heat exchanger

$$\varepsilon = \frac{1 - \exp[-NTU(1 + C_r)]}{1 + C_r}$$

$$NTU = -\frac{\ln[1 - \varepsilon(1 + C_r)]}{1 + C_r}$$

For *counterflow concentric tube* heat exchanger

$$\varepsilon = \frac{1 - \exp[-NTU(1 - C_r)]}{1 - C_r\exp[-NTU(1 - C_r)]} \qquad (C_r < 1)$$

$$\varepsilon = \frac{NTU}{1 + NTU} \qquad (C_r = 1)$$

$$NTU = \frac{1}{C_r - 1}\ln\left(\frac{\varepsilon - 1}{\varepsilon C_r - 1}\right) \qquad (C_r < 1)$$

$$NTU = \frac{\varepsilon}{1 - \varepsilon} \qquad (C_r = 1)$$

RADIATION

Types of Bodies

Any Body

For any body, $\alpha + \rho + \tau = 1$, where

α = absorptivity (ratio of energy absorbed to incident energy)
ρ = reflectivity (ratio of energy reflected to incident energy)
τ = transmissivity (ratio of energy transmitted to incident energy)

Opaque Body

For an opaque body: $\alpha + \rho = 1$

Gray Body

A gray body is one for which

$\alpha = \varepsilon$, $(0 < \alpha < 1; 0 < \varepsilon < 1)$, where

ε = the emissivity of the body

For a gray body: $\varepsilon + \rho = 1$

Real bodies are frequently approximated as gray bodies.

Black body

A black body is defined as one which absorbs all energy incident upon it. It also emits radiation at the maximum rate for a body of a particular size at a particular temperature. For such a body

$\alpha = \varepsilon = 1$

Shape Factor (View Factor, Configuration Factor) Relations

Reciprocity Relations

$A_i F_{ij} = A_j F_{ji}$, where

A_i = surface area (m^2) of surface i

F_{ij} = shape factor (view factor, configuration factor); fraction of the radiation leaving surface i that is intercepted by surface j; $0 \le F_{ij} \le 1$

Summation Rule for N Surfaces

$$\sum_{j=1}^{N} F_{ij} = 1$$

Net Energy Exchange by Radiation between Two Bodies

Body Small Compared to its Surroundings

$\dot{Q}_{12} = \varepsilon \sigma A \left(T_1^4 - T_2^4\right)$, where

$\dot{Q}_{12}$ = the net heat transfer rate from the body (W)

ε = the emissivity of the body

σ = the Stefan-Boltzmann constant [$\sigma = 5.67 \times 10^{-8}$ W/(m^2•K^4)]

A = the body surface area (m^2)

T_1 = the absolute temperature [K] of the body surface

T_2 = the absolute temperature [K] of the surroundings

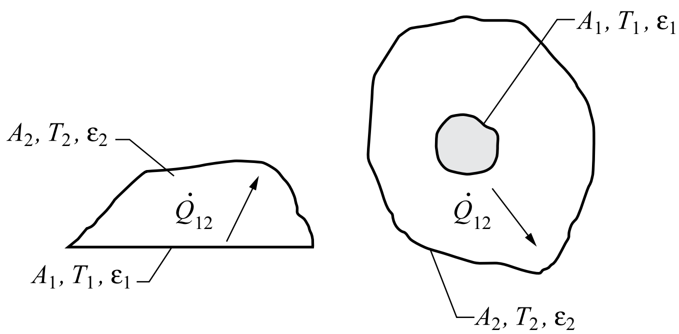
Net Energy Exchange by Radiation between Two Black Bodies

The net energy exchange by radiation between two black bodies that see each other is given by

$$\dot{Q}_{12} = A_1 F_{12} \sigma \left(T_1^4 - T_2^4\right)$$

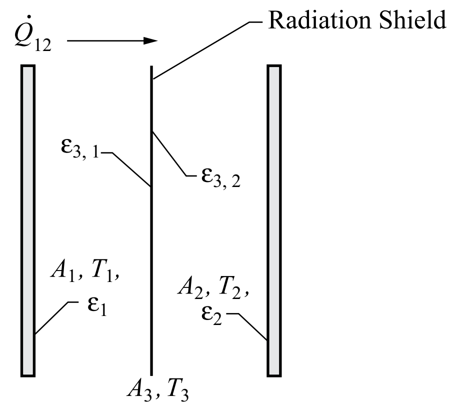
Net Energy Exchange by Radiation between Two Diffuse-Gray Surfaces that Form an Enclosure

Generalized Cases



$$\dot{Q}_{12} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

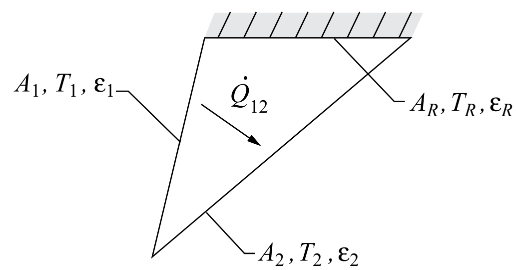
One-Dimensional Geometry with Thin Low-Emissivity Shield Inserted between Two Parallel Plates



$$\dot{Q}_{12} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{13}} + \dfrac{1-\varepsilon_{3,1}}{\varepsilon_{3,1} A_3} + \dfrac{1-\varepsilon_{3,2}}{\varepsilon_{3,2} A_3} + \dfrac{1}{A_3 F_{32}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

Reradiating Surface

Reradiating Surfaces are considered to be insulated or adiabatic $\left(\dot{Q}_R = 0\right)$.



$$\dot{Q}_{12} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{12} + \left[\left(\dfrac{1}{A_1 F_{1R}}\right) + \left(\dfrac{1}{A_2 F_{2R}}\right)\right]^{-1}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$